max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

**Uwe Waldmann**
**Christoph Weidenbach**

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**Tutorials for "Automated Reasoning"**
**Exercise sheet 4**

**Exercise 4.1:** (*3 P*)
Which of the following closed formulas are valid, satisfiable, unsatisfiable? Explain your decision.

a) $\forall x\, p(x) \rightarrow \exists x\, p(x)$

b) $\forall x\, (p(x) \rightarrow p(f(x))) \wedge p(b) \wedge \neg p(f(f(b)))$

c) $[\forall x\, (p(x) \rightarrow p(f(x))) \wedge p(b)] \rightarrow \forall x\, p(x)$

d) $\exists x\, p(x) \rightarrow p(b)$

e) $[\forall x\, (p(x) \rightarrow p(f(x)))] \rightarrow \exists x\, p(x)$

f) $[\forall x\, (p(x) \rightarrow p(f(x))) \wedge p(b)] \rightarrow \forall x\, \neg p(x)$

**Exercise 4.2:** (*3 P*)
Prove Proposition 2.4 from the lecture.

**Exercise 4.3:** (*2 P*)
Assume the homomorphic extension of a substitution where we only change the rule for quantifiers into $(Qx\, F)\sigma = Qx F\sigma$. Assuming this extension, give a counterexample for Proposition 2.4 from the lecture.

**Exercise 4.4:** (*2 P*)
Prove that for all sentences $F$, $\Sigma$-algebras $\mathcal{A}$ and valuations $\beta$:

$$\mathcal{A}, \beta \models F \Rightarrow \mathcal{A} \models F$$

**Challenge Problem:** (*2 Bonus Points*)
Give a first-order formula (without equality) that has only infinite models.

Submit your solution in lecture hall 003 during the lecture on May 22. Please write your name and the date of your tutorial group (Mon, Thu, Fri) on your solution.

**Note:** Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.